ONE-DIMENSIONAL EXPANSION OF A METEOR TRAIL

N. A. Arkabayev

ABSTRACT. A mathematical analysis is made of the initial stage of expansion of hot vapors and gases in a meteor trail.

Study of the initial stage of the expansion of hot vapors and gases trailing a meteoric body is of considerable interest. /34

The earth's atmosphere is quite rarefied at altitudes of 80 - 100 km, as we know, and its density is between 10^{-8} and 10^{-9} g/cm^3. Pressure is, for all practical purposes, zero.

Two approaches to the solution of problems of the discontinuum theory are possible: (1) using classical gas dynamics equations, and (2) using statistical physics methods.

The function defining particle density in phase space generally is taken from observations in applying statistical physics methods, so the solution is inexact.

We find it more desirable to use gas dynamics methods, including the interaction between the meteor and the atmospheric molecules surrounding it.

Let us consider the process of the expansion of the hot vapors and gases trailing a meteoric body, relative to the coordinate system associated with the body. In this system of coordinates, molecules vaporized from the meteoric body can be considered as molecules scattering from a cloud. And the mass of air in the trail has unit time momentum of

$$L = \Gamma \frac{S\rho'}{M} u^2, \qquad (1)$$

where

Γ is the drag coefficient;

S is the area of the front section of the meteoric body;

ρ' is the atmospheric density;

M is the mass of the meteoric body;

u is the velocity of the meteoric body upon invasion into the atmosphere.

The increase in the mass of the air in the trail in unit time is equal /35
to

$$b_1 = \lambda \frac{S\rho'}{2M} u^3, \qquad (2)$$

where

λ is the heat-transfer coefficient.

The drag coefficient, Γ, can be greater, or less, than unity at the same time that the heat-transfer coefficient, λ, always is less than (or equal to) unity.

The coefficients Γ and λ always are associated with each other in one way or another by physical processes which perform and accompany impulse and energy transfer.

Γ and λ are variables in the general cases, but the range of change is significantly less than the range of change in other magnitudes which determine the drag force and energy flow. Therefore, in the first approximation Γ and λ can be considered as constants.

The increase in the mass of the air in the trail in unit time, equated to a unit of mass of the meteoric body is equal to

$$b = \frac{b_1}{Q'} = \lambda \frac{S\rho'}{2MQ'} u^3, \qquad (3)$$

where

Q' is the energy necessary to heat and volatilize 1 gram of meteoric body material.

Let us consider the expansion of the trail as cylindrically symmetrical. The fundamental gas dynamic equations will, in the given case, then have the form

(1) impulse equation (Euler's)

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} = -\frac{1}{\rho} \frac{\partial P}{\partial r}, \qquad (4)$$

where

v is the radial velocity component;

ρ is the trail density;

P is the trail pressure;

(2) continuity equation

$$\frac{\partial \rho}{\partial t}+\frac{\partial(v\rho)}{\partial r}+\frac{\rho v}{r}=b\rho, \tag{5}$$

(3) energy equation

$$c_v T\,\frac{d\ln\sigma}{dt}=bQ', \tag{6}$$

where

$$\sigma=e^{\frac{s-s_0}{c_v}}, \tag{7}$$

s is the entropy per unit mass of the gas;

C_v is the specific heat of the gas for constant volume;

T is the trail temperature.

Let us take v, c, and ρ as dependent variables for convenience in subsequent calculations.

The equation of state for a polytropic gas

$$P = \sigma\rho^{k}, \tag{8}$$

where k is the polytropic index, leads to

$$\frac{dP}{\rho}=\left(\frac{\partial P}{\partial \rho}\right)d\ln\rho+\left(\frac{\partial P}{\partial \sigma}\right)\frac{d\sigma}{\rho}=cd\ln\rho+\frac{cd\ln\sigma}{k}.$$

Further, since

$$c = k\,\sigma\rho^{k-1},$$

then

$$d\ln\sigma=d\ln c-(k-1)d\ln\rho,$$

so

$$\frac{dP}{\rho}=\frac{1}{k}(dc+cd\ln\rho).$$

Because, in the case of a polytropic gas

$$dQ=c_v T d\ln\sigma=c_v T[d\ln c-(k-1)d\ln\rho],$$

Eq. (6), the conservation of energy equation, will have the form:

$$\frac{d\ln c}{dt}=(k-1)\frac{d\ln\rho}{dt}+\frac{1}{c_v T}bQ'... \tag{9}$$

From whence it follows that the basic set of equations for the variables, v, c, and ρ, can be written in the form

$$\left.\begin{aligned}\rho\left\{\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}+\frac{1}{k}\frac{\partial c}{\partial r}\right\}+\frac{c}{k}\frac{\partial\rho}{\partial r}=0,\\ \frac{\partial\rho}{\partial t}+v\frac{\partial\rho}{\partial r}+\rho\left(\frac{\partial v}{\partial r}+\frac{u}{r}\right)=b\rho,\\ \frac{1}{k(k-1)}\left(\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial r}\right)+\frac{c}{k}\left(\frac{\partial v}{\partial r}+\frac{v}{r}\right)=bQ'.\end{aligned}\right\} \tag{10}$$

/37

Let us expand the solutions of these equations in a power series near the coordinate origin

$$\left.\begin{aligned}\rho=\sum_{\lambda,\mu=0}^{\infty}\rho_{\lambda,\mu}r^{\lambda}t^{\mu},\\ c=\sum_{i,\kappa=0}^{\infty}c_{i,\kappa}r^{i}t^{\kappa},\\ v=\sum_{l,m=0}^{\infty}v_{lm}r^{l}t^{m}.\end{aligned}\right\} \tag{11}$$

Substituting the proposed solutions of Eq. (11) into the Eq. (10) set, we obtain

$$\sum_{\alpha,\beta=0}^{\infty}\sum_{\lambda,\mu=0}^{\alpha,\beta}\{(\beta-\mu+1)\rho_{\lambda,\mu}v_{\alpha-\lambda,\beta-\mu+1}+$$

$$+\sum_{i,\kappa=0}^{\lambda,\mu}(\lambda-i+1)v_{i,\kappa}\rho_{\alpha-\lambda,\beta-\mu}v_{\lambda-i+1,\mu-\kappa}+\frac{1}{k}[(\alpha-\lambda+$$

$$+1)\rho_{\lambda,\mu}c_{\alpha-\lambda+1,\beta-\mu}+(\lambda+1)\rho_{\lambda+1,\mu}c_{\alpha-\lambda,\beta-\mu}]\}r^{\alpha}t^{\beta}=0.$$

$$\Bigg(\sum_{\alpha,\beta=0}^{\infty}\{(\beta+1)\rho_{\alpha,\beta+1}+\sum_{\lambda,\mu=0}^{\alpha,\beta}[(\lambda+1)\rho_{\lambda+1,\mu}v_{\alpha-\lambda,\beta-\mu}+$$

$$+(\alpha-\lambda+1)\rho_{\lambda,\mu}v_{\alpha-\lambda+1,\beta-\mu}]\}+\sum_{\alpha=-1,\,\beta=0}^{\infty}\sum_{\lambda,\mu=0}^{\alpha+1,\beta}\rho_{\lambda,\mu}\times$$

$$\times v_{\alpha-\lambda+1,\beta-\mu})r^{\alpha}t^{\beta}=b\sum_{\alpha,\beta=0}^{\infty}\rho_{\alpha,\beta}r^{\alpha}t^{\beta}. \qquad /38$$

$$\Bigg(\sum_{\alpha,\beta=0}^{\infty}\left\{\frac{\beta+1}{k(k+1)}c_{\alpha,\beta+1}+\right.$$

$$+\sum_{\lambda,\mu=0}^{\alpha,\beta}\left[\frac{\lambda+1}{k(k+1)}\,c_{\lambda+1,\mu}\,v_{\alpha-\lambda,\beta-\mu}+\frac{1}{k}c_{\lambda,\mu}\times\right.$$

$$\left.\left.\times v_{\alpha-\lambda+1,\beta-\mu}\right]+\sum_{\alpha=-1,\beta=0}^{\infty}\sum_{\lambda,\mu=0}^{\alpha+1,\beta}c_{\lambda,\mu}v_{\alpha-\lambda+1,\beta-\mu}\Bigg)r^{\alpha}t^{\beta}=bQ'.$$

We have transformed differential equations (10) into a system of algebraic equations defining the coefficients of the Eq. (11) series.

REFERENCES

1. Levin, B. Yu., Fizicheskaya teoriya meteorov i meteornoye veshchestvo v solnechnoy sisteme [Physical Theory of Meteors and Meteoric Material in the Solar System], "Nauka" Press, 1956.

2. Stanyukovich, K. P., Neustanovivshiyesya dvizheniya sploshnoy sredy [Nonsteady Motion of a Continuum]. Moscow, 1955.